Interaction of criteria and Robust Ordinal Regression in Bi-polar PROMETHEE Methods

Salvatore Corrente[1], José Rui Figueira[2], Salvatore Greco[1]

[1]Department of Economics and Business, University of Catania, Italy
salvatore.corrente@unict.it, salgreco@unict.it
[2]CEG-IST, Instituto Superior Técnico, Av. Rovisco Pair, 1049-001 Lisboa, Portugal
figueira@ist.utl.pt

Abstract. In this paper we consider the bipolar approach to Multiple Criteria Decision Analysis (MCDA). In particular we aggregate positive and negative preferences by means of the bipolar PROMETHEE method. To elicit preferences we consider Robust Ordinal Regression (ROR) that has been recently proposed to derive robust conclusions through the use of the concepts of possible and necessary preferences. It permits to take into account the whole set of preference parameters compatible with the preference information given by the Decision Maker (DM).

Keywords: Multiple criteria outranking methods, Interaction between criteria, Bi-polar Choquet integral.

1 Introduction

Multiple Criteria Decision Analysis (MCDA) (for state-of-the-art surveys on MCDA see [5]) dealing with the comparison of the reasons in favor and against a preference of an alternative a over an alternative b is of the utmost importance. This kind of comparison is important, but it is only a part of the question. Indeed, after *recognizing the criteria in favor and the criteria against* of the preference of a over b, there is the very tricky question of comparing them (for a general discussion about bipolar aggregations of pros and cons in MCDA see [14]). In this second step, some important observations must be taken into account.

One element that should be considered is the *synergy* or the *redundancy* of criteria in favor of a preference of an action a against an action b. Of course there could be similar effects of synergy and redundancy regarding the criteria against the comprehensive preference of a over b. We have also to take into account the antagonism effects related to the fact that *the importance of criteria may also depend on the criteria which are opposed* to them. Those types of interactions between criteria have been already taken into consideration in the ELECTRE methods [13]. In this paper, we deal with the same problem using the bipolar Choquet integral [7, 8] (for the original Choquet integral see [4]) applied to the PROMETHEE method [3].

The paper is organized as follows. In the next section we introduce the application of the bipolar Choquet integral to PROMETHEE method. In the third section, we discuss elicitation of preference information permitting to fix the value of the preference parameter of the model (essentially the bicapacity of the bipolar Choquet integral). To take into account that there may be not only one, but a plurality of bicapacities representing the preference information, we propose also to adopt Robust Ordinal Regression (ROR) [12, 6, 1, 11, 10], in order to take into account the whole set of bicapacities compatible with the Decision Maker (DM) preferences. Within ROR we distinguish between necessary preference, in case an alternative a is at least as good as an alternative b for all the compatible bicapacities, and the possible preference, in case an alternative a is at least as good as an alternative b for at least one of the compatible bicapacities. The last section contains conclusions.

2 The Bipolar PROMETHEE

Let us consider a set of actions or alternatives $A = \{a, b, c, \ldots\}$ evaluated with respect to a set of criteria $G = \{g_1, \ldots, g_n\}$, where $g_j : A \to \mathbb{R}$, $j \in \mathcal{J} = \{1, \ldots, n\}$ and $|A| = m$. PROMETHEE [2, 3] is a well known MCDA method that aggregates preference information of a DM using an outranking relation. Considering a weight w_j representing the importance of criterion g_j within the family of criteria G, an indifference threshold q_j, and a preference threshold p_j, for each criterion g_j, PROMETHEE builds a non decreasing function $P_j(a, b)$ with respect to the difference $d_j(a, b) = g_j(a) - g_j(b)$, whose formulation (see [2] for other formulations) could be the following

$$P_j(a,b) = \begin{cases} 0 & \text{if } d_j(a,b) \leq q_j \\ \frac{d_j(a,b)-q_j}{p_j-q_j} & \text{if } q_j < d_j(a,b) < p_j \\ 1 & \text{if } d_j(a,b) \geq p_j \end{cases}$$

It represents the degree of preferability of a over b on criterion g_j.
For each ordered pair of alternatives $(a, b) \in A$, PROMETHEE method computes the value

$$\pi(a,b) = \sum_{j \in \mathcal{J}} w_j P_j(a,b)$$

representing how much alternative a is preferred to alternative b. It can assume values between 0 and 1 and obviously the greater the value of $\pi(a, b)$, the greater the preference of a over b is.
In order to compare an alternative a against all the other alternatives of the set A, PROMETHEE computes the positive and the negative net flow of a in the following way:

$$\phi^-(a) = \frac{1}{m-1} \sum_{c \in A \setminus \{a\}} \pi(c,a) \qquad \text{and} \qquad \phi^+(a) = \frac{1}{m-1} \sum_{c \in A \setminus \{a\}} \pi(a,c).$$

These net flows represent, respectively, how much the alternatives in $A \setminus \{a\}$ are preferred to a and how much a is preferred to the alternatives in $A \setminus \{a\}$. Besides the negative and the positive flows, PROMETHEE computes also the net flow $\phi(a) = \phi^+(a) - \phi^-(a)$. Taking into account these net flows, three relations can be built: preference ($\mathcal{P}$), indifference ($\mathcal{I}$), and incomparability ($\mathcal{R}$). In order to see how alternatives a and b are compared in PROMETHEE I method, see [2]; in case of PROMETHEE II, the comparison between alternatives a and b is done considering their net flows $\phi(a)$ and $\phi(b)$. In particular, we have that a is preferred to b if $\phi(a) > \phi(b)$, while a and b are indifferent if $\phi(a) = \phi(b)$.

Within the bipolar framework, the bipolar preference functions $P_j^B : A \times A \to [-1, 1], j \in \mathcal{J}$ are aggregated as follows

$$P_j^B(a,b) = P_j(a,b) - P_j(b,a) = \begin{cases} P_j(a,b) & \text{if } P_j(a,b) > 0 \\ \\ -P_j(b,a) & \text{if } P_j(a,b) = 0 \end{cases}$$

2.1 Determining comprehensive preferences

The aggregation of bipolar preference functions P_j^B through the bipolar Choquet integral is based on a bicapacity [7, 8], being a function $\hat{\mu} : P(\mathcal{J}) \to [-1, 1]$, where $P(\mathcal{J}) = \{(A, B) : A, B \subseteq \mathcal{J} \text{ and } A \cap B = \emptyset\}$, such that

- $\hat{\mu}(\emptyset, \mathcal{J}) = -1, \hat{\mu}(\mathcal{J}, \emptyset) = 1$, $\hat{\mu}(\emptyset, \emptyset) = 0$,
- for all $(A, B), (C, D) \in P(\mathcal{J})$, if $A \subseteq C$ and $B \supseteq D$, then $\hat{\mu}(A, B) \leq \hat{\mu}(C, D)$.

The interpretation of the bicapacity is the following: for $(A, B) \in P(\mathcal{J})$, and considering a pair of alternatives $(a, b) \in A \times A$, $\hat{\mu}(A, B)$ gives the net weight for the preference of a over b of criteria from A in favor of a and criteria from B in favor of b.

Given $(a, b) \in A \times A$, the bipolar Choquet integral of preference functions $P_j^B(a, b)$ representing the comprehensive preference of a over b with respect to the bicapacity $\hat{\mu}$ can be written as follows

$$Ch^B(P^B(a,b), \hat{\mu}) = \int_0^1 \hat{\mu}(\{j \in \mathcal{J} : P_j^B(a,b) > t\}, \{j \in \mathcal{J} : P_j^B(a,b) < -t\})dt.$$

Operationally, the bi-polar aggregation function $P_j^B(a, b)$ can be computed as follows. For all the criteria $j \in \mathcal{J}$, the absolute values of this function should be re-ordered in a non-decreasing way,

$$|P_{(1)}^B(a,b)| \leq |P_{(2)}^B(a,b)| \leq \ldots \leq |P_{(j)}^B(a,b)| \leq \ldots \leq |P_{(n)}^B(a,b)|$$

The comprehensive bi-polar Choquet integral with respect to the bicapacity $\hat{\mu}$ for the pair $(a, b) \in A \times A$ can now be determined as follows:

$$Ch^B(P^B(a,b), \hat{\mu}) = \sum_{j \in \mathcal{J}^>} |P_{(j)}^B(a,b)| \Big[\hat{\mu}(C_{(j-1)}, D_{(j-1)}) - \hat{\mu}(C_{(j)}, D_{(j)})\Big]$$

where:
$\hat{\mu}$ is a bi-capacity, $P^B(a,b) = \Big[P_j^B(a,b),\ j \in \mathcal{J}\Big]$, $C_{(0)} = \big\{j \in J : P_j^B(a,b) > 0\big\}$, $D_{(0)} = \big\{j \in J : P_j^B(a,b) < 0\big\}$, $\mathcal{J}^> = \{j \in \mathcal{J} \ : \ |P_{(j)}^B(a,b)| > 0\}$, $C_{(j)} = \{i \in \mathcal{J}^> \ : \ P_i^B(a,b) \geq |P_{(j)}^B(a,b)|\}$, and $D_{(j)} = \{i \in \mathcal{J}^> \ : \ -P_i^B(a,b) \geq |P_{(j)}^B(a,b)|\}$.

The value $Ch^B(P^B(a,b),\hat{\mu})$ gives the comprehensive preference of a over b and it is equivalent to $\pi(a,b) - \pi(b,a) = P^C(a,b)$ in the classical PROMETHEE method. Let us remark that it is reasonable to expect that $P^C(a,b) = -P^C(b,a)$. This leads to the following *symmetry condition*,

$$Ch^B(P^B(a,b),\hat{\mu}) = -Ch^B(P^B(b,a),\hat{\mu}).$$

Proposition 2.1 $Ch^B(P^B(a,b),\hat{\mu}) = -Ch^B(P^B(b,a),\hat{\mu})$, *for all possible* a,b, *iff* $\hat{\mu}(C,D) = -\hat{\mu}(D,C)$ *for each* $(C,D) \in P(\mathcal{J})$.

The above redefinition of $\pi(a,b) - \pi(b,a)$ in bi-polar terms leads to the following bi-polar definition of the net flows,

$$\phi^B(a) = \frac{1}{m-1} \sum_{c \in A \setminus \{a\}} Ch^B(P^B(a,c),\hat{\mu})$$

2.2 Determining the importance, the interaction, and the power of the opposing criteria

Several studies dealing with the determination of the relative importance of criteria were proposed in MCDA (see e.g. [17]). The question of the interaction between criteria was also studied in the context of MAUT methods [15]. In this section we present a quite similar methodology for outranking methods, which takes into account also the power of the opposing criteria.

2.3 The case of PROMETHEE method

The use of the bi-polar Choquet integral is based on a bi-polar capacity which assigns numerical values to each element $P(\mathcal{J})$. Let us remark that the number of elements of $P(\mathcal{J})$ is 3^n. This means that the definition of a bi-polar capacity requires a rather huge and unpractical number of parameters. Moreover, the interpretation of these parameters is not always simple for the DM. Therefore, the use of the bi-polar Choquet integral in real-world decision making problems requires some methodology to assist the DM in assessing the preference parameters (bi-polar capacities). In the following we consider only the $2-$order decomposable capacities, a particular class of bi-polar capacity.

2.4 Defining a manageable and meaningful bi-polar capacity measure

We define a $2-$order decomposable bi-capacity [9] such that for all
$(C,D) \in P(\mathcal{J})$

$$\hat{\mu}(C,D)=\mu^{+}(C,D)-\mu^{-}(C,D)$$

where

- $\mu^{+}(C,D)=\sum_{j\in C}a^{+}(\{j\},\emptyset)+\sum_{\{j,k\}\subseteq C}a^{+}(\{j,k\},\emptyset)+\sum_{j\in C,\ k\in D}a^{+}(\{j\},\{k\})$
- $\mu^{-}(C,D)=\sum_{j\in D}a^{-}(\emptyset,\{j\})+\sum_{\{j,k\}\subseteq D}a^{-}(\emptyset,\{j,k\})+\sum_{j\in D,\ k\in C}a^{-}(\{k\},\{j\})$

The interpretation of each $a^{\pm}(.)$ is the following:

- $a^{+}(\{j\},\emptyset)$, represents the power of criterion g_j by itself; this value is always positive.
- $a^{+}(\{j,k\},\emptyset)$, represents the interaction between g_j and g_k, when they are in favor of the preference of a over b; when its value is zero there is no interaction; on the contrary, when the value is positive there is a synergy effect when putting together g_j and g_k; a negative value means that the two criteria are redundant.
- $a^{+}(\{j\},\{k\})$, represents the power of criterion g_k against criterion g_j, when criterion g_j is in favor of a over b and g_k is against to the preference of a over b; this leads always to a reduction or no effect on the value of μ^{+} since this value is always non-positive.

An analogous interpretation can be applied to the values $a^{-}(\emptyset,\{j\})$, $a^{-}(\emptyset,\{j,k\})$, and $a^{-}(\{k\},\{j\})$.

In what follows, for the sake of simplicity, we will use a_j^{+}, a_{jk}^{+}, $a_{j|k}^{+}$, instead of $a^{+}(\{j\},\emptyset)$, $a^{+}(\{j,k\},\emptyset)$, and, $a^{+}(\{j\},\{k\})$, respectively; and a_j^{-}, a_{jk}^{-}, $a_{j|k}^{-}$, instead of $a^{-}(\emptyset,\{j\})$, $a^{-}(\emptyset,\{j,k\})$, and $a^{-}(\{k\},\{j\})$, respectively, obtaining

$$\hat{\mu}(C,D)=\mu^{+}(C,D)-\mu^{-}(C,D)=\sum_{j\in C}a_j^{+}-\sum_{j\in D}a_j^{-}+\sum_{\{j,k\}\subseteq C}a_{jk}^{+}-\sum_{\{j,k\}\subseteq D}a_{jk}^{-}+\sum_{j\in C,\ k\in D}a_{j|k}$$

where, $a_{j|k}=a_{j|k}^{+}-a_{j|k}^{-}$.
The following conditions should be fulfilled.
Monotonicity conditions

1) $\mu^{+}(C,D)\leq\mu^{+}(C\cup\{j\},D),\quad\forall\, j\in\mathcal{J},\ \forall(C\cup\{j\},D)\in P(\mathcal{J})$

$$\sum_{h\in C}a_h^{+}+\sum_{\{h,k\}\subseteq C}a_{hk}^{+}+\sum_{h\in C,k\in D}a_{h|k}^{+}\leq\sum_{h\in C\cup\{j\}}a_h^{+}+\sum_{\{h,k\}\subseteq C\cup\{j\}}a_{hk}^{+}+\sum_{h\in C\cup\{j\},k\in D}a_{h|k}^{+}\Leftrightarrow$$
$$\Leftrightarrow a_j^{+}+\sum_{k\in C}a_{jk}^{+}+\sum_{k\in D}a_{j|k}^{+}\geq 0,\quad\forall\, j\in\mathcal{J},\ \forall(C\cup\{j\},D)\in P(\mathcal{J})$$

2) $\mu^+(C,D) \geq \mu^+(C,D\cup\{j\}),\ \ \forall\, j \in \mathcal{J},\ \forall (C,D\cup\{j\}) \in P(\mathcal{J})$

$$\sum_{h\in C} a_h^+ + \sum_{\{h,k\}\subseteq C} a_{hk}^+ + \sum_{h\in C,k\in D} a_{h|k}^+ \geq \sum_{h\in C} a_h^+ + \sum_{\{h,k\}\subseteq C} a_{hk}^+ + \sum_{h\in C,k\in D\cup\{j\}} a_{h|k}^+ \Leftrightarrow$$
$$\Leftrightarrow \sum_{h\in C} a_{h|j}^+ \leq 0,\ \ \forall\, j \in \mathcal{J},\ \forall (C,D\cup\{j\}) \in P(\mathcal{J})$$

The same kind of monotonicity should be satisfied for μ^-. Let us call them Conditions 3) and 4). They are equivalent to the general monotonicity for μ^-, i.e.,

$$\forall\ (C,D),\ (E,F)\ \in\ P(\mathcal{J})\ \text{ such that }\ C\supseteq E,\ D\subseteq F,\ \ \mu^-(C,D)\leq \mu^-(E,F).$$

Conditions 1), 2), 3) and 4) together ensure the monotonicity of the bi-capacity, $\hat{\mu}$, on $\mathcal{J}$, obtained as the difference of μ^+ and μ^-, that is,

$$\forall\ (C,D),\ (E,F)\ \in\ P(\mathcal{J})\ \text{ such that }\ C\supseteq E,\ D\subseteq F,\ \ \hat{\mu}(C,D)\geq \hat{\mu}(E,F).$$

Boundary conditions

1. $\mu^+(\mathcal{J},\emptyset) = 1$, i.e., $\sum_{j\in\mathcal{J}} a_j^+ + \sum_{\{j,k\}\subseteq\mathcal{J}} a_{jk}^+ = 1$
2. $\mu^-(\emptyset,\mathcal{J}) = 1$, i.e., $\sum_{j\in\mathcal{J}} a_j^- + \sum_{\{j,k\}\subseteq\mathcal{J}} a_{jk}^- = 1$

2.5 The 2-order bi-polar Choquet integral

The following theorem gives a definition of the bi-polar Choquet integral in terms of the above 2-order decomposition.

Theorem 2.5 *If the bi-capacity $\hat{\mu}$ is $2-$order decomposable, then for all $x \in \mathbb{R}^n$*

$$\begin{aligned}
Ch^B(x,\hat{\mu}) = &\sum_{j\in\mathcal{J},x_j>0} a_j^+ x_j + \sum_{j\in\mathcal{J},x_j<0} a_j^- x_j + \\
&+ \sum_{j,k\in\mathcal{J},j\neq k,x_j,x_k>0} a_{jk}^+ \min\{x_k,x_j\} + \sum_{j,k\in\mathcal{J},j\neq k,x_j,x_k<0} a_{jk}^- \max\{x_k,x_j\} + \\
&\sum_{j,k\in\mathcal{J},x_j>0,x_k<0} a_{j|k}^+ \min\{x_j,-x_k\} + \sum_{j,k\in\mathcal{J},x_j>0,x_k<0} a_{j|k}^- \max\{-x_j,x_k\}
\end{aligned}$$

Proposition 2.5 *If $\hat{\mu}$ is 2-order decomposable then $\hat{\mu}(C,D) = -\hat{\mu}(D,C)$ for each $(C,D) \in P(\mathcal{J})$ iff*

1. for each $j \in \mathcal{J}$, $a_j^+ = a_j^- = a_j$,
2. for each $\{j,k\} \subseteq \mathcal{J}$, $a_{jk}^+ = a_{jk}^- = a_{jk}$,
3. for each $j,k \in \mathcal{J}$, $j \neq k$, $a_{j|k}^+ = a_{k|j}^-$.

2.6 Assessing the preference information

On the basis of the above 2−order decomposition and holding the symmetry condition in Proposition 2.5, we propose the following methodology which simplifies the assessment of the preference information. We consider the following information given by the DM and their representation in terms of linear constraints:

1. *Comparing pairs of actions.* The constraints represent some pairwise comparisons on a set of training actions. Given two actions a and b, the DM may prefer a to b, b to a or be indifferent to both:
   (a) the linear constraint associated with $a\mathcal{P}b$ is $Ch^B(P^B(a,b),\hat{\mu}) > 0$;
   (b) the linear constraint associated with $a\mathcal{I}b$ is $Ch^B(P^B(a,b),\hat{\mu}) = 0$.
2. *Comparison of the intensity of preferences between pairs of actions.* This comparison can be stated as follows:

$$Ch^B(P^B(a,b),\hat{\mu}) > Ch^B(P^B(c,d),\hat{\mu}) \quad \text{if} \quad (a,b)\mathcal{P}(c,d)$$

   where, $(a,b)\mathcal{P}(c,d)$ means that the comprehensive preference of a over b is larger than the comprehensive preference of c over d.
3. *Importance of criteria.* A partial ranking over the set of criteria $\mathcal{J}$ may be provided by the DM:
   (a) criterion g_j is more important than criterion g_k, which leads to the constraint $a_j > a_k$;
   (b) criterion g_j is equally important to criterion g_k, which leads to the constraint $a_j = a_k$.
4. *Interaction between pairs of criteria.* The DM can provide some information about interaction between criteria:
   (a) if the DM feels that interaction between g_j and g_k is more important than the interaction between g_p and g_q, the constraint should be defined as follows: $a_{jk} > a_{pq}$;
   (b) if the DM feels that interaction between g_j and g_k is the same of the interaction between g_p and g_q, the constraint will be the following: $a_{jk} = a_{pq}$.
5. *The sign of interactions.* The DM may be able, for certain cases, to provide the sign of some interactions. For example, if there is a synergy effect when criterion g_j interacts with criterion g_k, the following constraint should be added to the model: $a_{jk} > 0$.
6. *The power of the opposing criteria.* Concerning the power of the opposing criteria several situations may occur. For example:

(a) when the opposing power of g_k is larger than the opposing power of g_h, with respect to g_j, which expresses a positive preference, we can define the following constraint: $a^+_{j|k} > a^+_{j|h}$;
(b) if the opposing power of g_k, expressing negative preferences, is larger with g_j rather than with g_h, the constraint will be $a^+_{j|k} > a^+_{h|k}$.

2.7 A linear programming model

All the constraints presented in the previous section along with the symmetry, boundary and monotonicity conditions can now be put together and form a system of linear constraints. Strict inequalities can be converted into weak inequalities adding a variable ε. It is well-know that such a system has a feasible solution if and only if when maximizing ε, its value is strictly positive [15]. Considering constraints of Proposition 2.5, the linear programming model can be stated as follows (where $j\mathcal{P}k$ means that criterion g_j is more important than criterion g_k; the remaining relations have similar interpretation):

$$\text{Max } \varepsilon$$

$$\left.\begin{array}{ll}
Ch^B(P^B(a,b),\hat{\mu}) \geq \varepsilon \ \text{ if } \ a\mathcal{P}b, & Ch^B(P^B(a,b),\hat{\mu}) = 0 \ \text{ if } \ a\mathcal{I}b, \\
Ch^B(P^B(a,b),\hat{\mu}) \geq Ch^B(P^B(c,d),\hat{\mu}) + \varepsilon \ \text{ if } \ (a,b)\mathcal{P}(c,d), & Ch^B(P^B(a,b),\hat{\mu}) = Ch^B(P^B(c,d),\hat{\mu}) \ \text{ if } \ (a,b)\mathcal{I}(c,d), \\
a_j - a_k \geq \varepsilon \ \text{ if } \ j\mathcal{P}k, & a_j = a_k \ \text{ if } \ j\mathcal{I}k, \\
a_{jk} - a_{pq} \geq \varepsilon \ \text{ if } \ \{j,k\}\mathcal{P}\{p,q\}, & a_{jk} = a_{pq} \ \text{ if } \ \{j,k\}\mathcal{I}\{p,q\}, \\
a_{jk} \geq \varepsilon \ \text{ if there is synergy between criteria } j \text{ and } k, & \\
a_{jk} \leq -\varepsilon \ \text{ if there is redundancy between criteria } j \text{ and } k, & \\
a_{jk} = 0 \ \text{ if criteria } j \text{ and } k \text{ are not interacting}, & \\
\text{Power of the opposing criteria of the type 6:} & \\
a^+_{j|k} - a^+_{j|p} \geq \varepsilon, & a^-_{j|k} - a^-_{j|p} \geq \varepsilon, \\
a^+_{j|k} - a^+_{p|k} \geq \varepsilon, & a^-_{j|k} - a^-_{p|k} \geq \varepsilon, \\
a^+_{j|k} - a^-_{j|p} \geq \varepsilon, & \\
\text{Symmetry condition (point 3. of Proposition 2.5):} & \\
a^+_{j|k} = a^-_{k|j}, \ \ \forall\, j,k \ \in \ \mathcal{J}, & \\
\text{Boundary and monotonicity constraints:} & \\
\displaystyle\sum_{j \in \mathcal{J}} a_j + \sum_{\{j,k\} \subseteq \mathcal{J}} a_{jk} = 1, & \\
a_j \geq 0 \ \ \forall\, j \in \mathcal{J}, & a^+_{j|k},\ a^-_{j|k} \ \leq 0 \ \ \forall\, j,k \in \mathcal{J}, \\
a_j + \displaystyle\sum_{k \in C} a_{jk} + \sum_{k \in D} a^+_{j|k} \geq 0, \ \ \forall\, j \in \mathcal{J}, \ \forall (C \cup \{j\}, D) \in P(\mathcal{J}), & \\
a_j + \displaystyle\sum_{k \in D} a_{jk} + \sum_{k \in C} a^-_{k|j} \geq 0, \ \ \forall\, j \in \mathcal{J}, \ \forall (C, D \cup \{j\}) \in P(\mathcal{J}). &
\end{array}\right\} E^{A^R}$$

2.8 Restoring PROMETHEE

The condition which allows to restore PROMETHEE is the following:

1. $\forall j,k \in \mathcal{J}, \ \ a_{jk} = a^+_{j|k} = a^-_{j|k} = 0.$

If condition 1 is not satisfied and holds

2. $\forall j,k \in \mathcal{J}, a^{+}_{j|k} = a^{-}_{j|k} = 0$,

then the comprehensive preference of a over b is calculated as the difference between the Choquet integral of the positive preference and the Choquet integral of the negative preference, with a common capacity for the positive and the negative preferences, i.e. there exist a capacity $\mu : 2^{\mathcal{J}} \to [0,1]$, with $\mu(\emptyset) = 0$, $\mu(\mathcal{J}) = 1$, and $\mu(A) \leq \mu(B)$ for all $A \subseteq B \subseteq \mathcal{J}$, such that

$$Ch^B(P^B(a,b),\hat{\mu}) = \int_0^1 \mu(\{j \in \mathcal{J} : P_j^B(a,b) > t\})dt - \int_0^1 \mu(\{j \in \mathcal{J} : P_j^B(a,b) < -t\})dt.$$

We shall call this type of aggregation of preferences, the Choquet integral PROMETHEE method.
If neither 1. nor 2. are satisfied, then we have the Bipolar Choquet integral.

2.9 A constructive learning preference information elicitation process

The previous Conditions 1-2 suggest a proper way to deal with the linear programming model in order to assess the interactive bi-polar criteria coefficients. Indeed, it is very wise to try before to elicit weights concordant with the classic PROMETHEE method. If this is not possible, one can consider a PROMETHEE method which aggregates positive and negative preferences using the Choquet integral. If, by proceeding in this way, we are not able to represent the DM's preferences, we can take into account a more sophisticated aggregation procedure by using the bi-polar Choquet integral. This way to progress from the simplest to the most sophisticated models can be outlined in a four step procedure as follows,

1. Solve the linear programming model adding the constraint related to the previous Condition 1. If the model has a feasible solution with $\varepsilon > 0$, the obtained preferential parameters are concordant with the classical PROMETHEE method. Otherwise,
2. Solve the linear programming model adding Condition 2. If there is a solution with $\varepsilon > 0$, the information is concordant with the Choquet integral PROMETHEE method. Otherwise,
3. Solve the problem without any of the Conditions 1-2. A solution with $\varepsilon > 0$ means that the preferential information is concordant with the bi-polar Choquet integral PROMETHEE method. Otherwise,
4. We can try to help the DM by providing some information about inconsistent judgments, when it is the case, by using a similar constructive learning procedure proposed in [16].

In fact, in the linear programming model some of the constraints cannot be relaxed, that is, the basic properties of the model (symmetry, boundary and monotonicity constraints). The remaining constraints can lead to an unfeasible linear system which means that the DM provided inconsistent information about her/his preferences. The methods proposed in [16] can then be used in this context, providing to the DM some useful information about inconsistent judgements.

3 ROR applied to Bipolar PROMETHEE method

In above sections we dealt with the problem of finding a set of measures restoring preference information provided by the DM in case where multiple criteria evaluations are aggregated by Bipolar PROMETHEE outranking method. In this context it is meaningful to take into account the Robust Ordinal Regression (ROR) [12, 6, 1, 11, 10]. ROR is a family of MCDA methodologies recently developed, taking into account not only one model compatible with preference information provided by the DM, but the whole set of models compatible with preference information provided by the DM considering two preference relations: the weak *necessary* preference relation, for which alternative a is necessarily weakly preferred to alternative b if a is at least as good as b for all compatible models, and the weak *possible* preference relation, for which alternative a is possibly weakly preferred to alternative b if a is at least as good as b for at least one compatible model. In case of bi-polar PROMETHEE method, we can consider the necessary and the possible preference relations as follows:

- a is weakly possibly preferred to b, and we shall write $a \succsim^P b$, if $Ch^B(P^B(a,b),\hat{\mu}) \geq 0$ for at least one bi-capacity $\hat{\mu}$ compatible with the preference information given by the DM,
- a is weakly necessarily preferred to b, and we shall write $a \succsim^N b$, if $Ch^B(P^B(a,b),\hat{\mu}) \geq 0$ for all bi-capacity $\hat{\mu}$ compatible with the preference information given by the DM.

Given two alternatives $a, b \in A$, the set of constraints E^{A^R}, and considering the following sets of constraints,

$$\left.\begin{array}{l} Ch^B(P^B(a,b),\hat{\mu}) \geq 0 \\ E^{A^R} \end{array}\right\} E^P(a,b), \qquad \left.\begin{array}{l} Ch^B(P^B(b,a),\hat{\mu}) \geq \varepsilon \\ E^{A^R} \end{array}\right\} E^N(a,b),$$

the necessary and possible preference relations for the couple $(a,b) \in A \times A$, can be computed as follows:

- a is weakly possibly preferred to b iff $E^P(a,b)$ is feasible and $\varepsilon^* > 0$ where $\varepsilon^* = \max \varepsilon$ s.t. $E^P(a,b)$,
- a is weakly necessarily preferred to b iff $E^N(a,b)$ is infeasible or $\varepsilon^* \leq 0$ where $\varepsilon^* = \max \varepsilon$ s.t. $E^N(a,b)$.

4 Conclusions

The paper dealt with the aggregation of positive and negative preferences by means of the bipolar PROMETHEE method. ROR methodology has been proposed to derive robust conclusions through the use of the concepts of possible and necessary preferences. It permits to take into account the whole set of preference parameters compatible with the preference information given by the DM.

References

1. S. Angilella, S. Greco, and B. Matarazzo. Non-additive robust ordinal regression: A multiple criteria decision model based on the choquet integral. *European Journal of Operational Research*, 201(1):277–288, 2010.
2. J. Brans and B. Maréchal. PROMETHEE Methods. In J. Figueira, S. Greco, and M. Ehrgott, editors, *Multiple Criteria Decision Analysis: State of the Art Surveys*. Springer, Berlin, 2005.
3. J. Brans and Ph. Vincke. A preference ranking organisation method: The PROMETHEE method for MCDM. *Management Science*, 31(6):647–656, 1985.
4. G. Choquet. Theory of Capacities. *Annales de l'Institute Fourier*, 5:131–295, 1953/54.
5. J. Figueira, S. Greco, and M. Ehrgott. *Multiple Criteria Decision Analysis: State of the Art Surveys*. Springer, Berlin, 2005.
6. J.R. Figueira, S. Greco, and R. Słowiński. Building a set of additive value functions representing a reference preorder and intensities of preference: GRIP method. *European Journal of Operational Research*, 195(2):460–486, 2009.
7. M. Grabisch and C. Labreuche. Bi-capacities-I: definition, Möbius transform and interaction. *Fuzzy sets and systems*, 151(2):211–236, 2005.
8. M. Grabisch and C. Labreuche. Bi-capacities-II: the Choquet integral. *Fuzzy sets and systems*, 151(2):237–259, 2005.
9. S. Greco and J.R. Figueira. Dealing with interaction between bi-polar multiple criteria preferences in outranking methods. *Research Report 11-2003*, INESC-Coimbra, Portugal, 2003.
10. S. Greco, M. Kadzinski, and R. Słowiński. Extreme ranking analysis in Robust Ordinal Regression. *Omega*, 40(4):488–501, 2012.
11. S. Greco, M. Kadziński, V.Mousseau, and R. Słowiński. ELECTRE^{GKMS}: Robust Ordinal Regression for outranking methods. *European Journal of Operational Research*, 214(1):118–135, 2011.
12. S. Greco, V. Mousseau, and R. Słowiński. Ordinal regression revisited: multiple criteria ranking using a set of additive value functions. *European Journal of Operational Research*, 191(2):416–436, 2008.
13. S. Greco J.R. Figueira and B.Roy. Electre methods with interaction between criteria: An extension of the concordance index. *European Journal of Operational Research*, 199(2):478–495, 2009.
14. S. Greco M. Grabish and M.Pirlot. Bipolar and bivariate models in multicriteria decision analysis: Descriptive and constructive approaches. *International Journal of Intelligent Systems*, 23(9):930–969, 2008.
15. J. Marichal and M. Roubens. Determination of weights of interacting criteria from a reference set. *European Journal of Operational Research*, 124(3):641–650, 2000.

16. V. Mousseau, J. Figueira, L. Dias, C. Gomes da Silva, and J. Clímaco. Resolving inconsistencies among constraints on the parameters of an MCDA model. *European Journal of Operational Research*, 147:72–93, 2003.
17. B. Roy and V. Mousseau. A theoretical framework for analysing the notion of relative importance of criteria. *Journal of Multi-Criteria Decision Analysis*, 5:145–159, 1996.